Towards a classification of Minimal (τ-)Tilting Infinite Algebras

Kaveh Mousavand (Queen's University)

based on joint work with **Charles Paquette**

k: an algebraically closed field
A: a finite dimensional basic connected associative k-algebra
Ideal(A): set of all 2-sided ideals of A
τ **:** Auslander-Reiten translation

Definition: An A-module M is called

* ***Rigid*** if it has no self-extension;
* ***Tilting*** if it is rigid, has projective dimension less that 2, and |M|=|A|.
* ***τ-Rigid*** if Hom(M,τ(M))=0;
* ***τ-Tilting*** if it is τ-rigid and |M|=|A|.
* ***brick*** if Hom(M,M)=k.

Definition: An algebra A is called

* ***distributive*** if Ideal(A) is a distributive lattice;
* ***(τ-)tilting finite*** (resp. *brick finite*) if there are only finitely many isomorphism classes of basic (τ-)tilting modules (resp. bricks).

Definition: An algebra A is ***minimal (τ-)tilting infinite*** if A is (τ-)tilting infinite, but every proper quotient of A is (τ-)tilting finite.

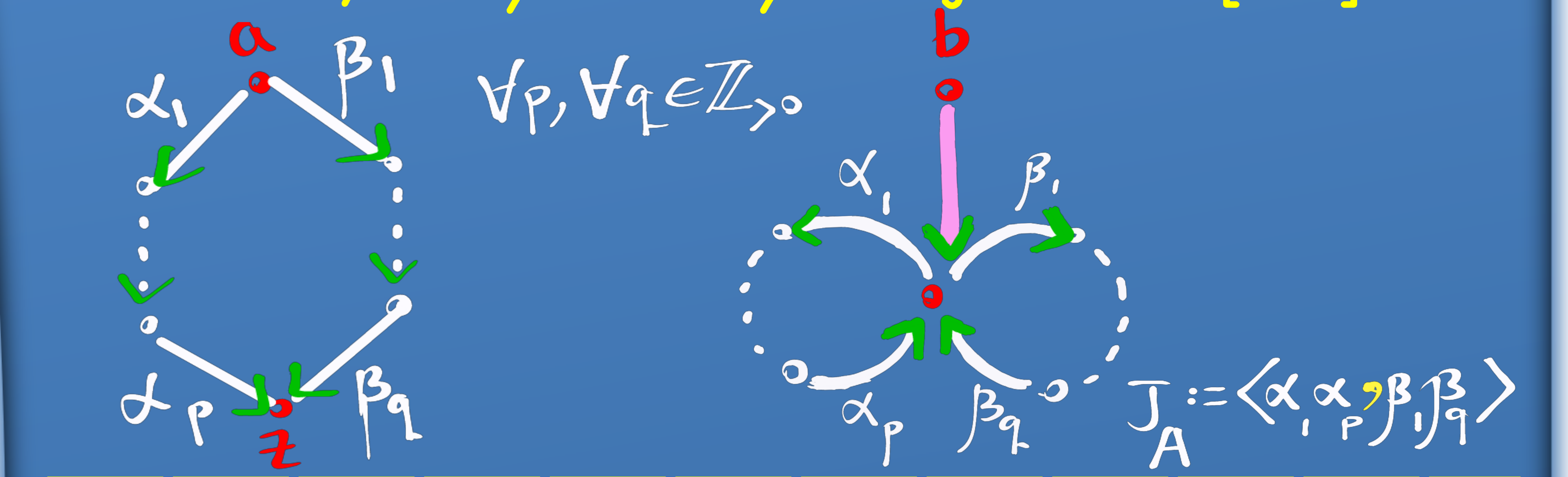
Remark: *All quotients of a τ-tilting finite algebra are τ-tilting finite;
** A tilting finite algebra may admit several tilting infinite quotients.

Theorem[DIJ]: *A is τ-tilting finite if and only if it is brick finite.*

Conjecture [Mo]: **A is τ-tilting infinite** if and only if **A admits a one-parameter family of bricks of the same length.**

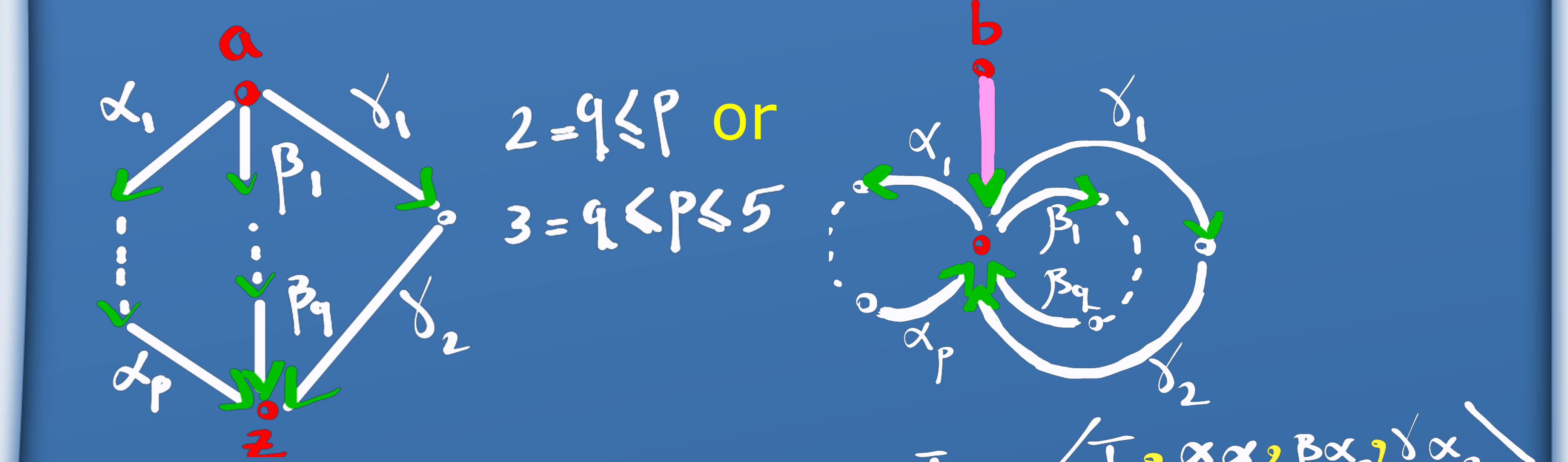
Theorem[MP1]: To prove Conjecture [Mo] it suffices to show it for the minimal τ-tilting infinite algebras with almost all bricks faithful.

Theorem [Mo, MP2]: The following algebras are **non-distributive minimal (τ-)tilting infinite.** Moreover, they satisfy Conjecture [Mo].



$\forall p, \forall q \in \mathbb{Z}_{>0}$

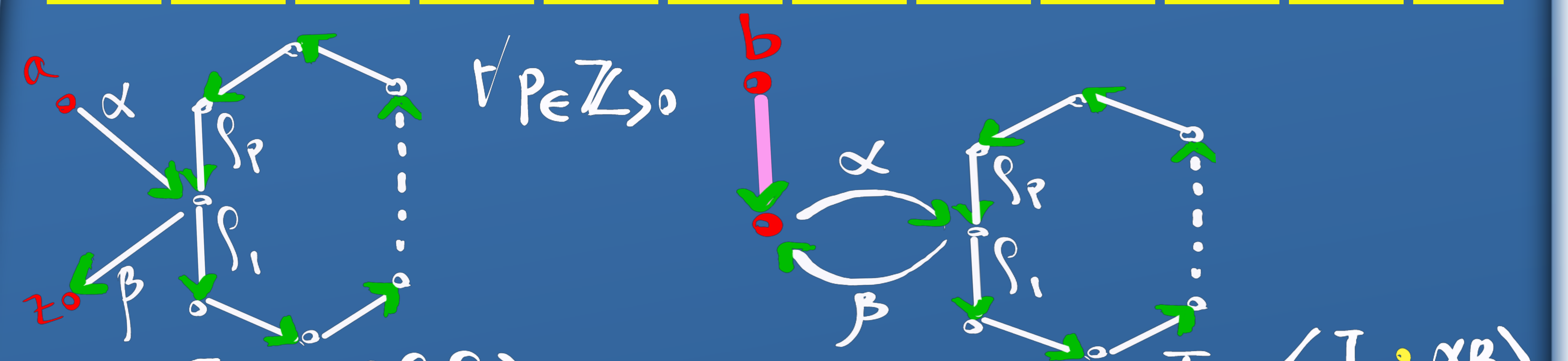
$J_A := \langle \alpha_1\alpha_p, \beta_1\beta_q \rangle$



$2 = q \leq p$ or $3 = q < p \leq 5$

$I_B := \langle \gamma_2\gamma_1 + \beta_q \cdots \beta_1 + \alpha_p \cdots \alpha_1 \rangle$

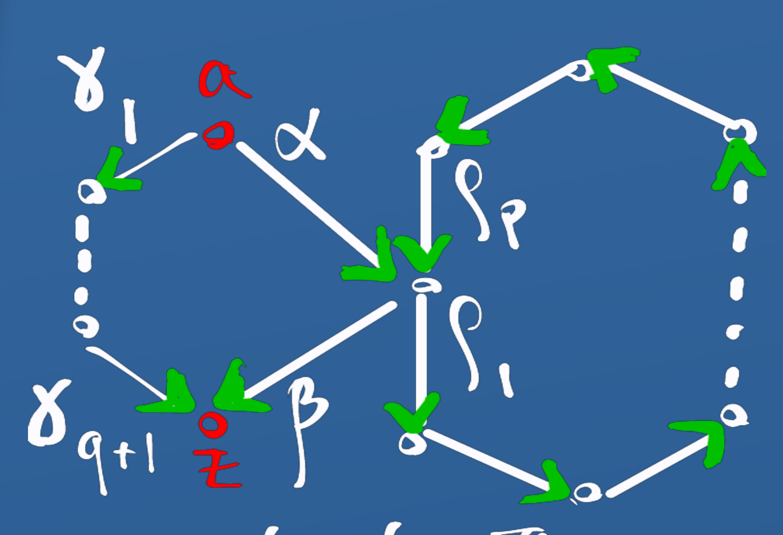
$J_B := \langle I_B, \alpha_1\alpha_p, \beta_1\alpha_p, \gamma_2\alpha_p, \alpha_1\gamma_2, \beta_1\gamma_2, \gamma_1\gamma_2, \alpha_1\beta_q, \beta_1\beta_q, \gamma_1\beta_q \rangle$



$\forall p \in \mathbb{Z}_{>0}$

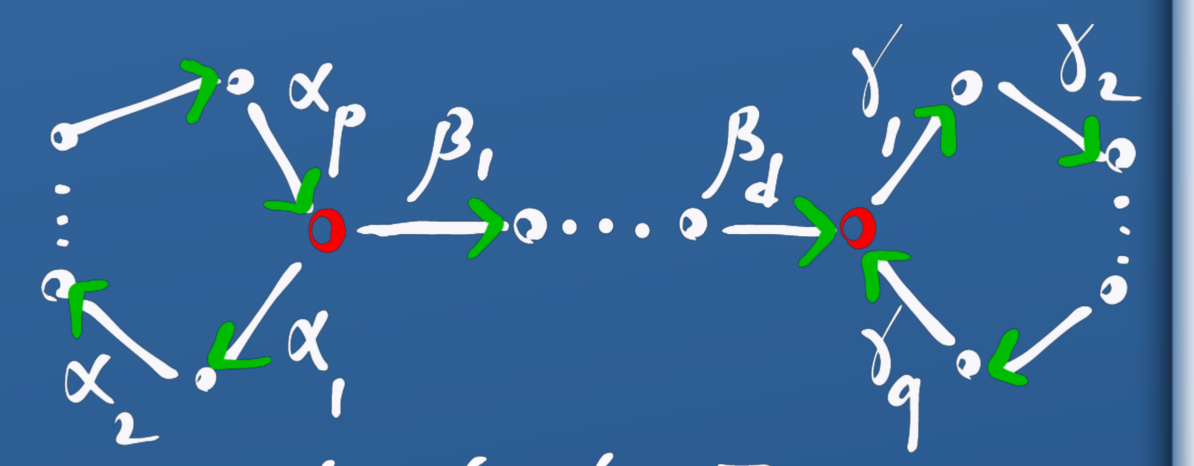
$I_C := \langle \rho_1\rho_p \rangle$

$J_C := \langle I_C, \alpha\beta \rangle$



$\forall p, \forall q \in \mathbb{Z}_{>0}$

$I_D := \langle \rho_1\rho_p, \gamma_{q+1}\cdots\gamma_1 - \beta\alpha \rangle$



$\forall p, \forall q, \forall d \in \mathbb{Z}_{>0}$

$I_E := \langle \alpha_1\alpha_p, \gamma_1\gamma_q \rangle$

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In addition to the notations and conventions from the previous page, let

$\mathcal{H}$: A hereditary abelain k-category

D^b($\mathcal{H}$): Bounded derived category of $\mathcal{H}$

mod-A: Category of finitely generated A-modules

Definition: An algebra A is ***piecewise hereditary*** of type $\mathcal{H}$ if D^b(mod-A) is triangle equivalent to D^b($\mathcal{H}$).

Theorem [MP2]: Every piecewise hereditary algebra A satisfies Conjecture [Mo].
Namely, A is τ-tilting infinite if and only if it admits one parameter family of bricks of the same length.

Definition: An algebra is called ***tilted*** if it is the endomorphism algebra of a tilting module over a path algebra A=kQ.

Corollary [MP2]: Tilted (and cluster-tilted) algebras satisfy Conjecture [Mo].
In fact, if A is tilted (or cluster-tilted), then A is representation infinite if and only if it admits one parameter family of bricks of the same length.

Corollary [MP2]: The minimal (τ-)tilting infinite algebras which are tilted are those explicitly given by Happel and Vossieck in [HV].

Closing Remarks:

* Conjecture [Mo] gives a geometric interpretation of the algebraic notion of tau-tilting finiteness.

** Conjecture [Mo] has also appeared in [SST], in the independent work of Schroll,Treffinger,Valdivieso.

References:

[DIJ] L. Demonet, O.Iyama, G.Jasso,
τ-tilting finite algebras, bricks and g-vectors.

[Mo] K. Mousavand,
τ-tilting finiteness of non-distributive algebras and their module varieties.

[MP1] K. Mousavand, C. Paquette,
Minimal (τ-)tilting infinite algebras.

[MP2] K. Mousavand, C. Paquette,
Some explicit families of minimal (τ-)tilting infinite algebras (In preparation).

[HV] D. Happel, D. Vossieck,
Minimal algebras of infinite representation type with preprojective component.

[SST] S. Schroll, H. Treffinger, Y. Valdivieso,
On band modules and τ-tilting finiteness.